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1- "A Design Study of a Numerically Controlled Frame Bending Machine", RINA. (UK-1965), Shama, M. A. and Miller, N. S., (90%)

2- "Plastic Bending of Short Mild Steel Beams", Bull. of the Faculty of Engineering, Alexandria University, (Egypt-1965), Shama, M. A. (100%)

3- "Behavior of Short Mild Steel Beams Bent into the Plastic Range of the Material" Bull. of the Faculty of Eng. Alexandria University, (Egypt-1967), Shama, M. A., (100%)

4- "The Impact of Application of a Numerically Controlled Plate Forming Machine on Shipyard Production" Bull of the Faculty of Engineering, Alexandria University, (Egypt-1968), Shama, M. A., 100%)

5- "Numerical Control of Plate Forming and Associated Problems", Shipbuilding and Shipping Record, Jan. (UK-1970), Shama, M. A., (100%)

6- " On the Calculation of Cold Forming Residual Stresses", Bull., Of the Faculty of Engineering, Alexandria University, Vol. XI. (Egypt-1970), Shama, M. A., (100%)

7- "Cold Forming Residual Stresses and Their Effect on the Accuracy of Post – Forming Operations", European Shipbuilding, No.2, and No. 3, Vol. 19. (Norway-1970) Shama, M. A., (100%)

8- "On the Calculation of Cold Forming Residual Stresses", Schiff und Hafen, (Germany-1974), Shama, M. A., (100%)

On the Calculation of Cold Forming Residual Stresses

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Summary

A method is given for calculating the magnitude and distribution of cold forming residual stresses. The method is based on the exponential idealization of the stress-strain diagram of the material and takes into account the deformed shape of the cross-section.

The method is used to investigate the effects of the relevant parameters on the magnitude and distribution of cold forming residual stresses for rectangular section beams. It is shown that the degree of bend and section deformation, resulting from plastic bending, have a marked influence on the magnitude and distribution of cold forming residual stresses.

Introduction

Cold forming of plates and sections is widely used in several industries, particularly in the shipbuilding industry, as it is potentially the most economic method used for forming ship plates and frames. This method of forming raises several problems, among them is residual stresses [1].

Residual stresses may also result from several other sources, such as welding, gas cutting, etc. [2]. These residual stresses when superimposed may weaken the fatigue resistance of the material, may have an adverse effect on the yield strength, may impair the buckling strength of columns [3], and may also promote the initiation and propagation of brittle fracture [4].

Apart from structural strength considerations, cold forming residual stresses can give rise to considerable distortions during the prefabrication and assembly stages of a ship [1]. In the shipbuilding industry, the elimination of these distortions has great economic advantages.

In order to assess their adverse effect, cold forming residual stresses should be accurately determined in terms of their magnitude and distribution. In reference [5], the effects of the shape of stress-strain diagram and section particulars on the magnitude and distribution of cold forming residual stresses are investigated and discussed.

In this paper an attempt is made to calculate the magnitude and distribution of cold forming residual stresses using the exponential idealization of the stress-strain diagram of the material, and also the deformed shape of the cross section. The method is applied to rectangular section beams to investigate the effects of the degree of bend and the deformation of the cross section on the magnitude and distribution of cold forming residual stresses.

List of Symbols

The following symbols and notation are used in the paper:

A	total cross sectional area
a	constant
d	depth of rectangular section beam
e	true strain
e_θ, e_R, e_z	principal strains in the tangential, radial and transverse directions respectively
h	half depth of rectangular section
h_c, h_t	distances of extreme fibre, in the compression and tension regions respectively, from neutral axis
K	degree of bend ($K = \rho_o/d$)
M	bending moment
N.A.	neutral axis
n	constant
R	radius of curvature at a depth y from N.A.
t_o	original thickness of rectangular section beam
t	thickness
t_c, t_t	thickness of section in the compression and tension regions, after plastic bending, at distances ± y from N.A.
x	distance from centroidal axis
y	distance from N.A.
$\bar{y}$	distance of centroid of an element from N.A.
σ	true stress
$(\sigma_e)_x$	elastic stress at a distance x from the centroidal axis
σ_y	yield stress
$(\sigma_b)_y$	bending stress at a depth y from N.A.
$(\sigma_r)_y$	residual stress at a depth y from N.A.
σ_θ, σ_R, σ_z	principal stresses in the tangential, radial and transverse directions respectively
ϵ	conventional strain
ϵ_y	yield strain
δ	distance between centroidal and neutral axes, ($\delta = y - x$)
η_t, η_c	numerical factors
θ	an angle representing the degree of bend
ρ	radius of curvature attained by N.A.
ρ_o, ρ_i	outer and inner radii of curvature

Method of Calculation

The magnitude and distribution of cold forming residual stresses depend on the magnitude and distribution of the bending stress (σ_b) and the elastic unloading stress (σ_e). The calculation of σ_b and σ_e depends on the assumed shape of the stress-strain diagram, shape of cross-section and the degree of bend [5].

In the following analysis, the calculation of cold forming residual stresses is based on the true shape of the stress-strain diagram and takes into account the section deformation resulting from plastic bending.

It is assumed that the true shape of the stress-strain diagram could be represented by [6]:

$$\sigma = \pm a \cdot e^{\frac{1}{n}} \qquad (1$$

where $e = \log_e(1+\epsilon)$; σ and e = true stress and strain respectively; a and n are constants depending on the material used.

For shipbuilding steel, the mean values of a and n are as follows:

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$a \approx$ 65.0 kg/sq.mm.
$n \approx$ 5.4

The calculation of cold forming residual stresses is summarised in the following steps:

a. Bending Stress Distribution

The bending stress distribution across the depth of any section is directly obtained from the stress-strain relationship as follows:

$$\sigma_b = \pm a \cdot e^{1/n} \quad (2)$$

where $e = \log(R/\rho)$.

The $\pm$ sign indicates tensile or compressive stresses, and therefore depends on the direction of bending.

b. Elastic Unloading Stress Distribution

The unloading stress distribution is calculated using the simple beam theory as follows:

$$(\sigma_e)_x = \frac{M}{I} \cdot x \quad (3)$$

where $x = y - \delta$ = distance from centroidal axis; δ = distance between centroidal and neutral axes; M = bending moment; and is calculated as follows:

$$M = \int_0^A \sigma_b \cdot y \cdot dA = \int_0^A a \cdot e^{1/n} \cdot y \cdot dA \quad (4)$$

where A = total area of the section.

The bending moment could be calculated as follows:

$$M = \sum_0^{A_t} a \cdot e^{\frac{1}{n}} \cdot \bar{y}_t \cdot dA_t + \sum_0^{A_c} a \cdot e^{\frac{1}{n}} \cdot \bar{y}_c \cdot dA_c \quad (5)$$

where $\bar{y}_t$ and $\bar{y}_c$ are the distances of the centroids of dA_t and dA_c respectively from N.A.; dA_t and dA_c are elementary areas in the tension and compression regions respectively and are given by:

$$dA_t = t_t \cdot dy$$

and $\quad (6)$

$$dA_c = t_c \cdot dy$$

where t_c and t_t are the thickness of the element dy in the compression and tension regions respectively.

Substituting (6) into (5) we get:

$$M = \sum_0^{h_t} a \cdot e^{\frac{1}{n}} \cdot \bar{y}_t \cdot t_t \cdot dy + \sum_{-h_c}^{0} a \cdot e^{\frac{1}{n}} \cdot \bar{y}_c \cdot t_c \cdot dy \quad (7)$$

In the case of flanged sections such as T-section, O.B.P. or an angle, which are normally used in welded ship constructions, the web and the flange are treated separately.

c. Residual Stress Distribution

The residual stress distribution, across the depth of any section, is the resultant of the bending and unloading stress distributions, and therefore is given by:

$$(\sigma_r)_y = \pm (\sigma_b)_y \mp (\sigma_e)_x \quad (8)$$

where $(\sigma_r)_y$ = residual stress at a depth y from N.A.

Substituting for $(\sigma_b)_y$ from (2) and $(\sigma_e)_x$ from (3) into (8), we get:

$$(\sigma_r)_y = \pm a \cdot e^{1/n} \mp \frac{M}{I}(y - \delta) \quad (9)$$

where M is calculated from expression (7).

Expression (9) gives the distribution of cold forming re[...]al stresses across the depth of any section.

Residual Stress Distribution Across the Depth of a Rectangular Section

The above general method is used to calculate the dist[...]tion of cold forming residual stresses across the depth of a [...]tangular section beam of depth d and thickness t.

Due to plastic bending, the beam thickness will increa[...] the compression region and will decrease in the tension re[...] The variation in the thickness across the depth of the secti[...] calculated in Appendix (I) and is given by:

$$t_c = \eta_c \cdot t_o.$$
$$t_t = \eta_t \cdot t_o.$$

where:

$$\eta_c = \left(\frac{R}{\rho}\right)^p$$
$$\eta_t = \left(\frac{R}{\rho}\right)^q$$

and:

$$p = -\frac{1 + 2\log\left(\frac{\rho_i}{R}\right)}{2 + \log\left(\frac{\rho_i}{R}\right)}$$

$$q = \frac{1}{3\log\left(\frac{\rho_o}{R}\right) - 2}$$

Substituting (10) into (7) we get:

$$M = a \cdot t_o \left[\sum_0^{h_t} e^{1/n} \cdot \eta_t \cdot \bar{y}_t \cdot \Delta y_t + \sum_{-h_c}^{0} e^{1/n} \cdot \eta_c \cdot \bar{y}_c \cdot \Delta y_c\right]$$

where Δy = an element of depth Δy and at a distance $\bar{y}$ [...] N.A.; h_t and h_c are the distances of extreme fibre on the [...]sion and compression regions respectively from N.A.

The computation of M could be performed by dividin[...] distances h_t and h_c respectively into m equally spaced inter[...]

Hence: $(\bar{y}_t)_i = \Delta y_t \cdot (i - \frac{1}{2})$

and $(\bar{y}_c)_i = \Delta y_c \cdot (i - \frac{1}{2})$

where $i = 1, 2, 3 \ldots m$

$$\Delta y_t = \frac{h_t}{m}$$

and $$\Delta y_c = \frac{h_c}{m}$$

For a rectangular section beam, it could be assumed tha[...] effect of thickness variation on the magnitude of the se[...] moment of area of the section is negligible, i.e., the second [...]ment of area is given by:

$$I \approx \frac{t_o \cdot d^3}{12}$$

Substituting for M and I from (12) and (13) respective[...] (9), we get:

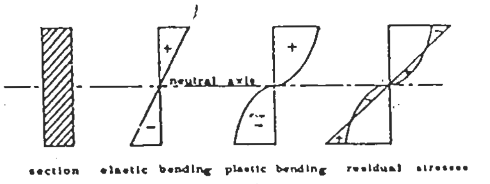
$$(\sigma_r)_y = \pm a \cdot e^{\frac{1}{n}} \mp \frac{12a}{d^3}\left[\sum_0^{h_t} e^{\frac{1}{n}} \cdot \eta_t \cdot \bar{y}_t \cdot \Delta y_t + \sum_{-h_c}^{0} e^{\frac{1}{n}} \cdot \eta_c \cdot \bar{y}_c \cdot \Delta y_c\right]$$

It is clear from expression (14) that, for rectangular section beams, the cold forming residual stress distribution does not depend on the magnitude of the beam thickness, but it depends on the variation in thickness across the beam depth.

The above method of calculation is used to compute the residual stress distribution for rectangular section beams having the following characteristics:

1. The depth of section varies from 5.0 cm to 20.0 cm every 5.0 cm.
2. The degree of bend varies from 2.0 to 20.0 every 2.0.

Fig. 1



The degree of bend is given by K, where $K = \rho_0/d$ and ρ_0 = radius of curvature at the convex side of the bent beam.

Results of Calculations

The results of the above calculations are given and analysed in terms of:

1. Effect of the degree of bend on the magnitude and distribution of cold forming residual stresses.
2. Effect of plastic deformation of a rectangular section on the magnitude and distribution of cold forming residual stresses.

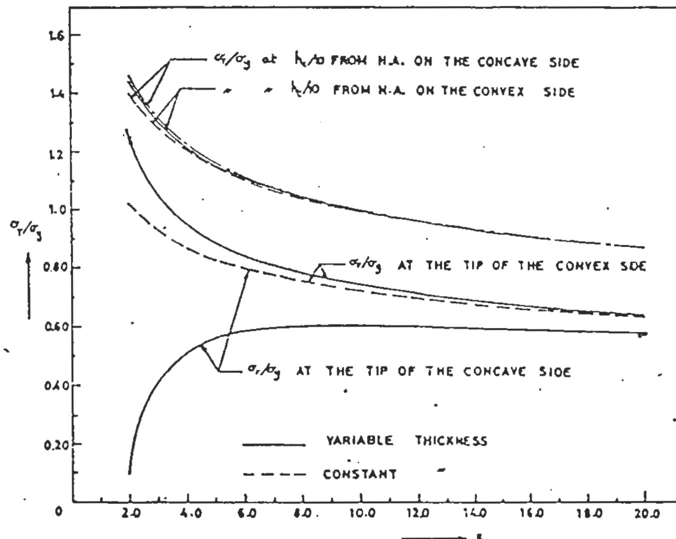


Fig. 2

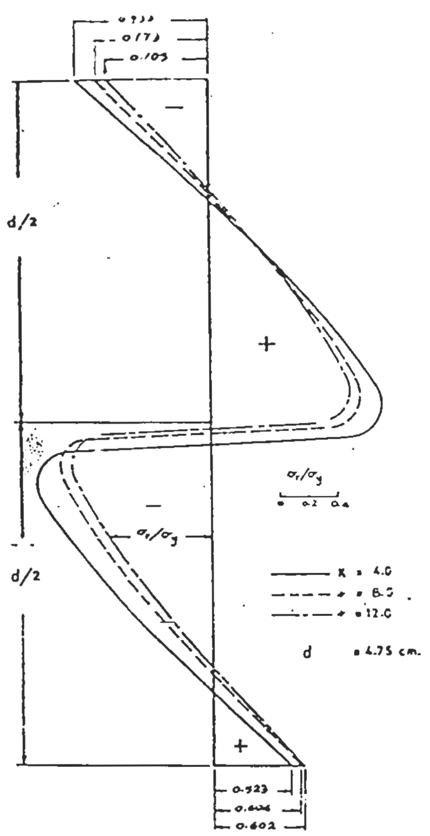
These two items are considered in some detail:

1. Effect of degree of bend.

Fig. (2) shows the effect of degree of bend on the magnitude of residual stresses at the extreme fibre in the tension and compression regions and also at a distance of approximately ± 5 % of the beam depth from the neutral axis of the bent beam.

Fig. (3) shows the effect of degree of bend on the distribution of residual stresses across the beam depth.

Fig. 3



From these figures, it is clear that when $2 < k \leqslant 8$ the degree of bend has a remarkable influence on the magnitude and distribution of residual stresses. When $k \geqslant 8$, the degree of bend has a small effect on residual stresses.

The effect of degree of bend on the magnitude and distribution of residual stresses for a beam having a depth = 10.0 cm is given quantitatively in the following table:

Table I

Convex side				Concave side			
k = 4.0		k = 12.0		k = 4.0		k = 12.0	
y_t	σ_r/σ_y	y_t	σ_r/σ_y	y_c	σ_r/σ_y	y_c	σ_r/σ_y
0.5358	+1.212	0.5108	+0.965	0.4641	−1.219	0.4891	−0.968
1.0717	+1.092	1.0217	+0.874	0.9282	−1.143	0.9782	−0.887
1.6076	+0.898	1.5326	+0.724	1.3923	−0.995	1.4673	−0.749
2.1435	+0.670	2.0435	+0.547	1.8564	−0.814	1.9564	−0.584
2.6794	+0.423	2.5543	+0.355	2.3205	−0.615	2.4456	−0.404
3.2153	+0.163	3.0652	+0.153	2.7846	−0.404	2.9347	−0.214
3.7512	−0.106	3.5761	−0.056	3.2487	−0.184	3.4238	−0.016
4.2871	−0.382	4.0870	−0.271	3.7128	+0.042	3.9129	+0.186
4.8230	−0.663	4.5978	−0.490	4.1769	+0.274	4.4021	+0.393
5.3589	−0.949	5.1087	−0.713	4.6410	+0.509	4.9812	+0.603

2. Effect of thickness variation

Due to plastic bending, the beam thickness will no longe be constant but will undergo a reduction, in the tension regior and an increase, in the compression region. This variation i thickness will certainly affect the position of the neutral ax of the section.

Fig. (4) shows the effect of the degree of bend on the pos tion of the neutral axis for different depths of the rectangul section. These curves are represented in Fig. (5) in a non-dime sional form.

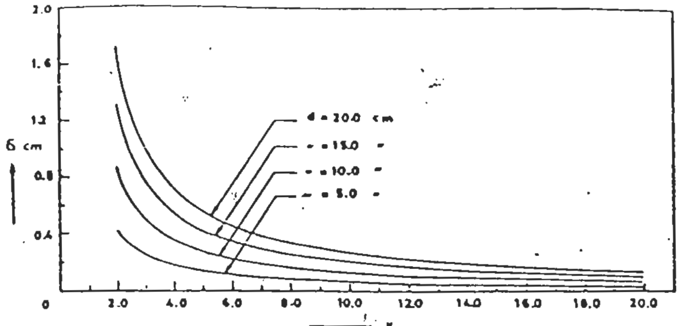


Fig. 4

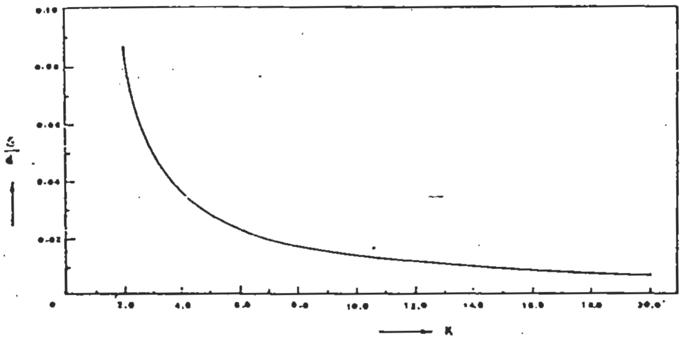


Fig. 5

Fig. (2) shows the effect of thickness variation on the magnitude of residual stresses at the tip and near the neutral axis.

Fig. (6) shows the effect of thickness variation on the distribution of residual stresses across the depth of section.

It is shown that the thickness variation has also a significant influence on the magnitude and distribution of residual stresses and on the position of the neutral axis of the section.

For a rectangular section beam having a depth d and bent to a sharp curvature given by k, the neutral axis position and tip residual stresses can be obtained from Figs. (5) and (2) respectively.

Conclusions

From the foregoing results and analysis, it is concluded that the degree of bend and the plastic deformation of the cross-section have a marked influence on the magnitude and distribution of residual stresses and also on the position of the neutral axis of the section.

It should be emphasised that the accurate assessment of the magnitude and distribution of cold forming residual stresses may help predicting the anticipated distortions resulting from any of the post-forming operations. These expected distortions could then be taken into account in the forming stage so as to improve the accuracy of the prefabricated units, and subsequently reduce assembly times and rectification costs.

Appendix I

Variation of thickness across the depth of section.

Due to plastic bending, the beam thickness will increase in the compression region and decrease in the tension region. In order to determine this variation in thickness, which is assumed to be due to bending action only, the following empirical relations are used [7]:

a. Condition of constancy of volume:
i.e. $de_\theta + de_R + de_z = 0$

b. The relationship between the principal stresses and stra[ins] in the plastic stage:

$$\text{i.e.} \quad \frac{d\sigma_\theta - d\sigma_R}{de_\theta - de_R} = \frac{d\sigma_\theta - d\sigma_z}{de_\theta - de_z} = \frac{d\sigma_R - d\sigma_z}{de_R - de_z}$$

In addition to (a) and (b), the following equilibrium c[on]ditions should be satisfied:

$$\Sigma F_R = 0 \text{ and } \Sigma F_T = 0$$

where F_R and F_T are the forces acting in the radial and tang[en]tial directions respectively.

It is assumed that only welded ship constructions are c[on]sidered, i.e. only deep sections of the plate type are dealt w[ith] (e.g. off-set bulb plate (O.B.P.), T-section, angle or flat bar).

It should also be noted that, since we are dealing with re[la]tively deep sections, a plane stress condition is assumed. F[ur]ther, if it is assumed that the ratios between the princi[pal] stresses remain constant [8] during loading and also that t[he] principal directions remain fixed during the deformation, [ex]pressions (1) and (2) could be simplified to:

$$e_\theta + e_R + e_z = 0$$

$$\text{and} \quad \frac{\sigma_\theta - \sigma_R}{e_\theta - e_R} = \frac{\sigma_\theta - \sigma_z}{e_\theta - e_z} = \frac{\sigma_R - \sigma_z}{e_R - e_z}$$

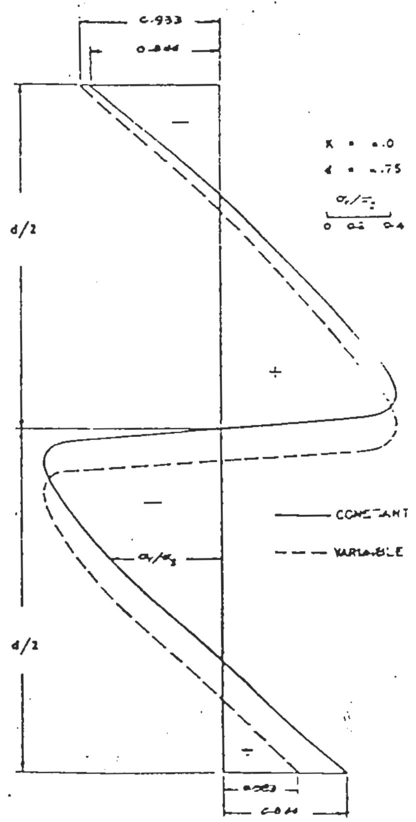


Fig. 6

i.e. total plastic strains are used instead of incremental strains

where $e_z = \log(t/t_0)$ (i)

$e_\theta = \log(R/\rho)$ (ii)

The calculation of the variation in thickness will be treated separately for the tension and compression regions, using the above assumptions and relations.

1. Variation in Thickness across the Tension Region

In this case, it is assumed that the web of the section is subjected to the tensile bending stresses, i.e. σ_θ is tensile.

Since there is no lateral loading, i.e. $\sigma_z = 0$, expression (5) is simplified as follows:

$$\frac{\sigma_\theta - \sigma_R}{e_\theta - e_R} = \frac{\sigma_\theta}{e_\theta - e_z} = \frac{\sigma_R}{e_R - e_z} \qquad (6)$$

Substituting e_R from (4) into (6) we get:

$$\frac{\sigma_R}{\sigma_R + \sigma_\theta} = \frac{e_\theta + 2\,e_z}{3\,e_z} \qquad (7)$$

In order to determine e_z from expression (7), the relationship between σ_R and σ_θ should be determined. This could be achieved by considering the equilibrium of an element in the vertical direction, see Fig. (7), which should satisfy the condition:

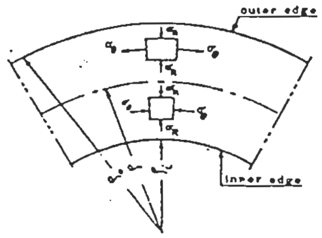


Fig. 7

$$\Sigma F_R = 0$$

which gives:

$$(\sigma_R + \sigma_\theta)\,dR = -R \cdot d\sigma_R \qquad (8)$$

Assuming that Tresca yield criterion is valid (8), the condition of yielding of material is given by:

$$\sigma_\theta + \sigma_R = \sigma_y \qquad (9)$$

Substituting condition (9) into (8), we get:

$$\sigma_y \cdot dR = -d\sigma_R \cdot R$$

i.e. $\log(\frac{\rho_o}{R}) = \sigma_R/\sigma_y$

Hence:

$$\sigma_R = \sigma_y \cdot \log(\frac{\rho_o}{R}) \qquad (10)$$

or:

$$\sigma_R = (\sigma_R + \sigma_\theta) \cdot \log(\frac{\rho_o}{R}) \qquad (11)$$

Substituting for $\frac{\sigma_R + \sigma_\theta}{\sigma_R}$ from (11) into (7), we get:

$$e_z = \frac{e_\theta}{3\,(\log\frac{\rho_o}{R}) - 2} \qquad (12)$$

Substituting for e_z and e_θ from (i) and (ii) into (12), we get:

$$t_t = t_0 \cdot (\frac{R}{\rho})^q \qquad ($$

where:

$$q = \frac{1}{3\,(\log\frac{\rho_o}{R}) - 2} \qquad ($$

Expression (13) gives the thickness at any depth y from N in the tension region and could be simplified by:

$$t_t = \eta_t \cdot t_o$$

where η_t = numerical factor representing the reduction thickness.

$$\eta_t = (R/\rho)^q \qquad ($$

2. Variation in Thickness across the Compression Region

In this case, it is assumed that the web of the section is su jected to the compressive bending stresses, i.e. σ_θ is compr sive.

The equilibrium of an element in the compression regic i.e. $\Sigma F_R = 0$ gives

$$(\sigma_R - \sigma_\theta)\,dR = -R \cdot d\sigma_R \qquad (1$$

Assuming that Tresca yield criterion is also valid (8), t condition of yielding of material is given by:

$$\sigma_\theta - \sigma_R = \sigma_y \qquad (1$$

Substituting for $(\sigma_\theta - \sigma_R)$ from (17) into (16), we get:

$$\sigma_y \cdot dR = R \cdot d\sigma_R .$$

Hence:

$$\sigma_R = \sigma_y \cdot \log(\frac{R}{\rho_i}) \qquad (1$$

or:

$$\sigma_R = (\sigma_\theta - \sigma_R) \cdot \log(\frac{R}{\rho_i}) \qquad (1$$

Expression (18), or (19), gives the distribution of radi stresses across the depth of the compression region.

Expression (6) could be written as follows:

$$\frac{\sigma_\theta - \sigma_R}{\sigma_R} = \frac{e_\theta - e_R}{e_R - e_z}$$

Substituting for e_R from condition (4) and for $\frac{\sigma_\theta - \sigma_R}{\sigma_R}$ fro (19), we get:

$$\log(\frac{R}{\rho_i}) = \frac{-\log\frac{R}{\rho} - 2\,(\log\frac{t}{t_o})}{2\,(\log\frac{R}{\rho}) + \log(\frac{t}{t_o})}$$

i.e.

$$t_c = t_o \cdot (\frac{R}{\rho})^p \qquad (2C$$

where:

$$p = -\frac{2\,(\log\frac{R}{\rho_i}) + 1}{\log(\frac{R}{\rho_i}) + 2} \qquad (2$$

Expression (20) gives the thickness at any depth y fro N.A. in the compression region and could be simplified by:

$$t_c = \eta_c \cdot t_o$$

where η_c = numerical factor representing the increase in thic ness

$$\eta_c = (\frac{R}{\rho})^p \qquad (2$$

Calculation of the Position of Neutral Axis

The position of neutral axis could be determined by equating the radial stresses at the neutral axis for both the tension and compression regions.

From Appendix (I), the radial stresses in the tension and compression regions are given by:

$$(\sigma_R)_t = \sigma_y \cdot \log(\frac{\rho_o}{R}) \qquad (a)$$

$$(\sigma_R)_c = \sigma_y \cdot \log(\frac{R}{\rho_i}) \qquad (b)$$

where $(\sigma_R)_t$ and $(\sigma_R)_c$ are rhe radial stresses in the tension and compression regions respectively.

Since at $R = \rho$, $(\sigma_R)_t$ and $(\sigma_R)_c$ should be equal, then we have:

$$\log(\frac{\rho_o}{\rho}) = \log(\frac{\rho}{\rho_i})$$

i.e.

$$\rho = \sqrt{\rho_i \cdot \rho_o}$$

Therefore, for a rectangular section beam of depth d, and outer radius ρ_o, we have:

[illegible]

$$h_t \approx \rho_o - \rho$$

$$h_c \approx \rho - \rho_i$$

References

[1] Shama, M. A., Miller, N.: "A design study of a numeric trolled frame bending machine". RINA, January, 1967.

[2] Welding in Shipbuilding. A symposium by the Institute ding, (1961), London.

[3] Yang, C. H., Beedle, L. S., and Johnston, B. G.: "Residu and the yield strength of steel beams". Progress report Welding Research Supplement, The Welding Journal, April 205.

[4] Yoshinki, M.: "Review of research work in large tankers out in Japan". ISSC, 1961, Glasgow.

[5] Shama, M. A.: "Cold forming residual stresses and their eff the accuracy of post-forming operations". European Shipbuilding, No. 2 and No. 3, Vol. XIX, 1970.

[6] Shanley, F. R.: "Strength of Materials". McGraw-Hill Book Co. Inc., New York, 1957.

[7] Lubahn, J. D., Sachs, G.: "Bending an ideal plastic metal". TASME, 1950.

[8] Wemper, S., Storn, Becket: "An introduction to the mech of deformable bodies". Prentice Hall, 1961.